Making sentential identity explicit through justification logic

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January 20, 2025

Non-Fregean logics proposed by Roman Suszko in *Abolition of the Fregean Axiom* are a system with propositional identity ($\equiv$), a connective stronger than equivalence ($\leftrightarrow$). Suszko argued that sentences denote situations, unlike classically, where they denote their truth values, thereby challenging what is known as the Fregean axiom and shedding light on the nature of identity. Suszko hinted at an intuitionistic version of SCI which was later proposed by Łukowski and called ISCI. The intuitionistic extension of a non-Fregean logic is that more interesting, as through its intuitionistic component it is linked with provability and the Brouwer-Heyting-Kolmogorov semantic (BHK). Similarly, through the notion of provability, Justification Logic (JL) was created, stemming from Logic of Proof (LP) proposed by Artemov. In Justification Logic one can operate on justifications explicitly, unlike in modal S4, for example the modal principle $\Box(A \to B) \to (\Box A \to \Box B)$ becomes the formula $t : (A \to B) \to (u : A \to [t \cdot u] : B)$, where a justification term $[t \cdot u]$ denotes a combination of justifications t and u. Going further, both ISCI and JL are hyperintensional and both deal with this fact in ways different than *adding layers* to the possible world semantic. Classically any true sentence p will be seen as equal to any other true sentence q, because both denote the same object – truth. In JL they are seen as equal at the propositional level: $p \leftrightarrow q$ holds, but different when it comes to justifications: $t : p \leftrightarrow t : q$ is not necessarily true. Same for non-Fregean logics: sentences p and q, even when equal when it comes to their truth value ($p \leftrightarrow q$ holds), may differ when it comes to the context they denote ($p \equiv q$ fails).

In the presentation I intend to *gently* introduce the notion of hyperintensionality through examples grounded in the cognitive paradigm, then show how exactly it unites JL and SCI. Next I shall give a brief presentation of a Gödel-Gentzen style translation for SCI and ISCI and explain what it tells us about identity. Finally, I shall share intuitions regarding the final goal of the thesis, propositional indentity expressed through justifications: some similarities in sequent calculi for LP and SCI; and how the BHK-interpretation for identity paired with lambda calculus is to guide us through the dense

forest of logical investigation.